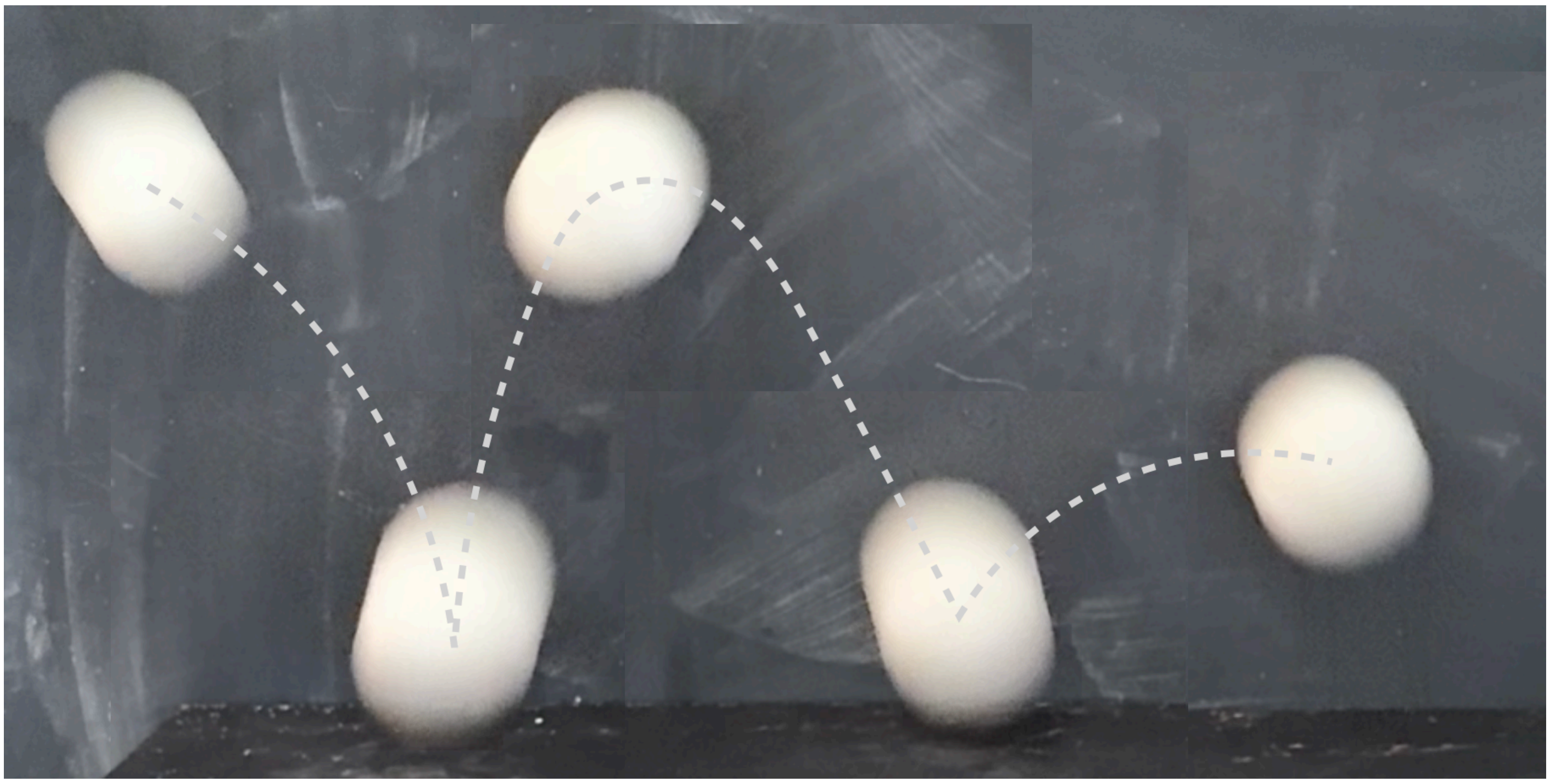
THE WORK ENERGY PRINCIPLE



Take a ball and bounce it along the floor. As it moves to the side, it doesn't get quite as high as it did on the previous bounce. How do we account for that motion? One way is to examine the situation using ideas of energy.

Side Note: Everything I learned about energy was from Bruce Sherwood and Ruth Chabay and their awesome textbook: Matter and Interactions (Wiley). Just letting you know, that's an awesome book.

This is kind of a big deal. In introductory physics, we really show you two different methods to describe interactions. The first method uses forces. Forces are interactions between two objects that can change the motion (change is key) of at least one of those objects. This is described with Newton's second law.

$$\vec{F}_{\text{net}} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t}$$

Hopefully, that equation isn't new. But I do want to point out some important features of this equation.

- It's a vector equation. Both the net force and the velocity are vectors. You knew that already, but I'm just trying to be clear.
- The velocity vector changes with time. This is a "time-based" expression.

ENERGY STUFF

Maybe you already know somethings about energy. You know there is energy stored in your smart phone battery. You know that it takes energy for you to move around during the day. Finally, you know that you have to pay someone to get energy to your house.

But now we need to think about energy from a physics perspective. In this chapter, we are going to look at energy, and how to change the energy of a system. In short, the work-energy principle says that the work done on a system is equal to its change in energy.

$$W = \Delta E$$

Yes, there's a lot more to do.

THE VECTOR DOT PRODUCT

Before we get to the fun stuff, we need some mathematical tools. In this case, it's the dot product (also called the scalar product). If you have two vectors($\overrightarrow{A}$ and $\overrightarrow{B}$) the dot product is a measure of the amount the two vectors are pointing in the same direction.

Before getting into that, let me give a vector translation guide. Here are three ways to represent the same vector.

$$\overrightarrow{A} = < 1,2,0 >$$

$$\overrightarrow{A} = 1\hat{x} + 2\hat{y}$$

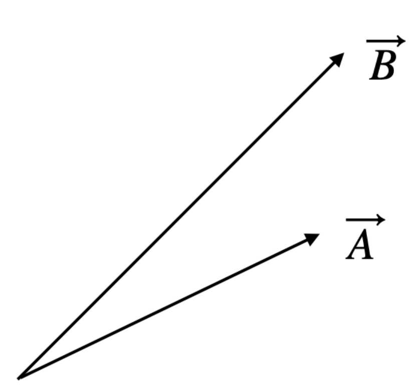
$$\overrightarrow{A} = 1\hat{i} + 2\hat{j}$$

Personally, I like the first method. It's just more compact. Yes, all three vectors could be considered "3D" with the last two just having a zero z component.

Now we can get to the dot product. Imagine I have the following two vectors.

$\overrightarrow{A} = < 2,1,0 >$

$\overrightarrow{B} = < 3,2,0 >$



We can calculate $\overrightarrow{A} \cdot \overrightarrow{B}$ as:

$$\overrightarrow{A} \cdot \overrightarrow{B} = A_x B_x + A_y B_y + A_z B_z$$

You just take the like-components of each vector, multiply them and then add them all up. For our example, it would be:

$\overrightarrow{A} \cdot \overrightarrow{B} = 2 \times 3 + 1 \times 2 = 6 + 2 = 8$

I know that might be confusing, but it's just an operation that we are going to use. You don't have to completely understand it. There are some other important dot product things you need to know.

1. The dot product commutes. $\overrightarrow{A} \cdot \overrightarrow{B} = \overrightarrow{B} \cdot \overrightarrow{A}$

2. There's a scalar version of this product. $\overrightarrow{A} \cdot \overrightarrow{B} = |\overrightarrow{A}||\overrightarrow{B}|\cos\theta$ where θ is the angle between the two vectors.

3. The dot product has units. Yes, the examples above were unitless. However, since it's a type of multiplication the result would have the product of units of the two vectors.

4. The dot product can be positive, negative or zero. It will be zero when the two vectors are perpendicular. If the two vectors have an angle less than 90 degrees, the dot product will be positive. For angles greater than 90, the dot product is negative.

Done.

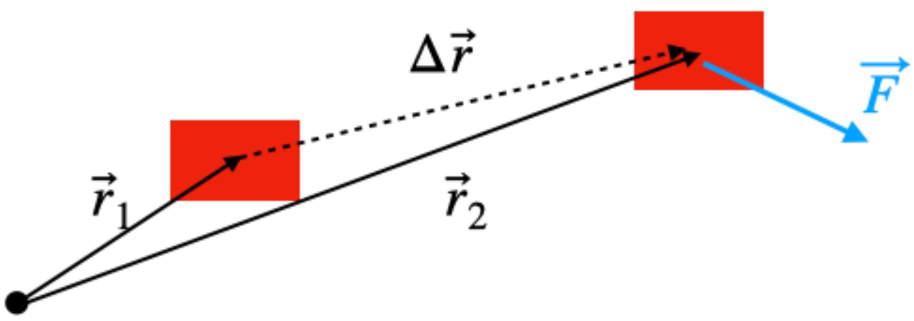
WORK BY A FORCE

The work done on a system is a way to either add to take away energy. In short, the work depends on the applied force and the displacement of the force. Oh wait, it's the dot product.

$$W = \overrightarrow{F} \cdot \Delta\vec{r} = F\Delta r \cos\theta$$

If the force is in units of Newtons and the displacement is in meters, then the work done is a Newton*meter which is equal to a Joule (the unit for energy). Let's just do a quick example.

A box starts at the vector position $\vec{r}_1 = < 1,1,0 >$ meters and moves to $\vec{r}_2 = < 3,2,0 >$ m with a force of $\overrightarrow{F} = < 2, -1,0 >$ Newtons pushing on it. Here's a picture.



The first step to calculating the work is find the value for $\Delta\vec{r}$.

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Delta\vec{r} = < 3,2,0 > - < 1,1,0 > = < 3,1,0 > \text{ meters}$$

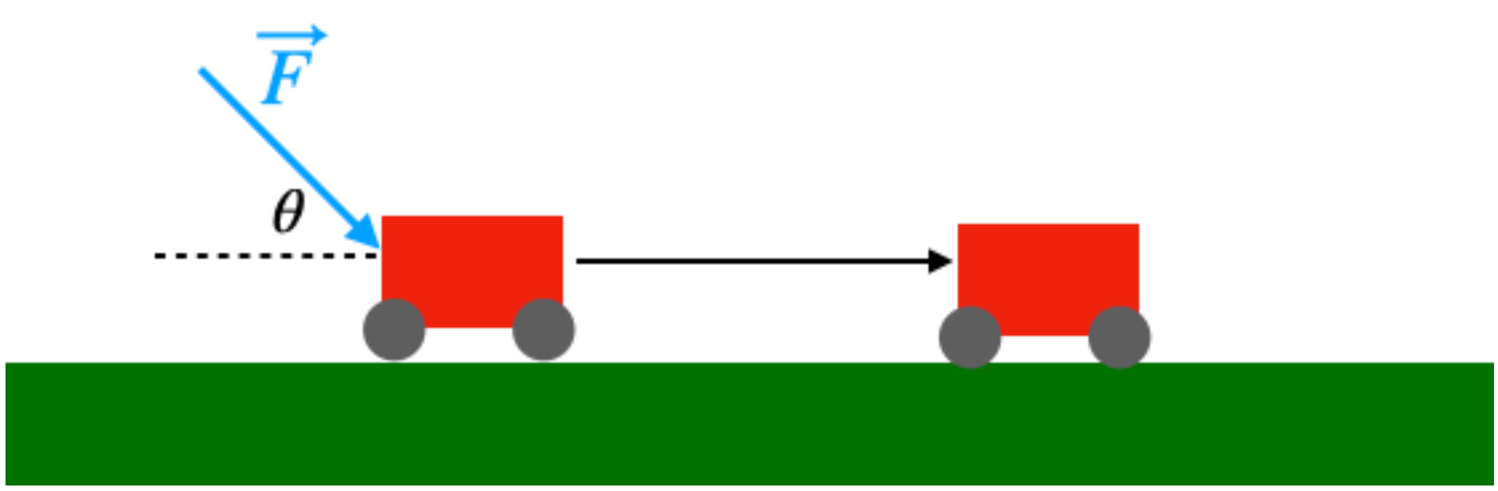
Remember that vector displacements with vector components are fairly straightforward. You just subtract the like-components.

Now we can take the dot product between $\overrightarrow{F}$ and $\Delta\vec{r}$.

$\overrightarrow{F} \cdot \Delta\vec{r} = < 2, -1,0 > \cdot < 3,1,0 > = 2 \times 3 - 1 \times 1 = 5$ Joules.

Done.

Here's another example. Bob pushes a lawn mower so that it moves 3 meters on a level yard. He pushes with a force of 40 Newtons at a downward angle such that it's 35 degrees above the horizontal.



In this situation, we don't have vectors in component form. That's fine. We can use the other definition of the dot product to calculate the work.

$$W = F\Delta r \cos\theta = (40\text{ N})(3\text{ m})\cos(35^\circ) = 98.3\text{ J}$$

Nice. Notice that since the mower is still sort of pushed in the direction of travel, the work done is positive. Also, this work calculation by itself doesn't say anything about the motion of mower. It's possible that it could speed up, move at a constant speed or even slow down depending on the other interactions involved.

WORK ENERGY FOR A POINT PARTICLE

Don't act like you've never seen or used a point particle. It's what you used for Newton's second law. You drew a force diagram showing all the forces on the object, but you never cared WHERE the forces where applied. That's because your object was just a point—it had no size. Oh sure, a tennis ball isn't a point. The box sliding down a plane isn't a point. Even the lawn mower above isn't a point. Still, this idealization of a point particle makes problems way easier to deal with.

Don't worry, we will look at non-point particles later.

The work-energy principle deals with systems. We will get into this more later, but for now consider a system that ONLY consists of a point. In that case, our work-energy principle says the following.

$$W = \Delta E = \Delta K$$

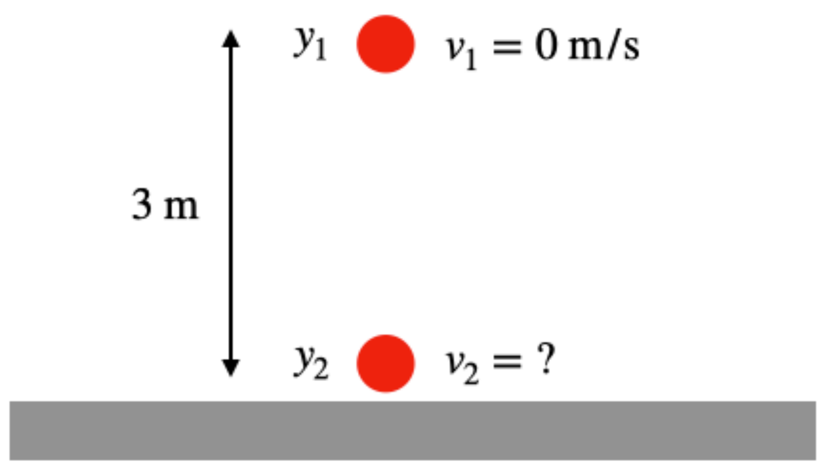
$$K = \frac{1}{2}mv^2$$

Here, K is the kinetic energy. It clearly depends on both the mass (m) and speed (v) of the particle. But notice that since mass is positive and the velocity is squared, this must be a positive quantity. Oh wait! Don't confuse K with ΔK. Yes, the change in kinetic energy can be positive, negative, or zero. One more quick point—technically v is a vector. You can't square a vector. So, what we do is to first take the magnitude of that vector and then square it.

If we have a system of just a point particle, it can only have kinetic energy. Don't worry, we will look at other systems later. Let's do a problem.

DROPPING BALL EXAMPLE

A ball with a mass of 100 grams is released from rest h = 3 meters above the floor. How fast is it moving right before it hits the ground?



The first step with the work-energy principle is to choose your system. In this case, I'm going with the system that only consists of the ball. This choice means two things. First, there can only be kinetic energy for the system. Second, there is one external interaction that will do work on the system—the gravitational interaction.

Let's calculate the work done by the gravitational force. There are two ways to do this. For the first method, I'm going to write both the gravitational force and the displacement as vectors and then calculate the dot product. For the displacement, I'm going to pick the floor as y = 0 meters.

$$\vec{F}_g = < 0, -mg, 0 >$$

$$\Delta\vec{r} = < 0,0,0 > - < 0,h,0 > = < 0,-h,0 >$$

$$W = \vec{F} \cdot \Delta\vec{r} = < 0,-mg,0 > \cdot < 0,-h,0 > = 0 + (-h)(-mg) + 0 = mgh$$

Notice that both the force and the displacement have negative y-components. This means that the dot product is going to be a positive value. Now for the second method, let's use the other definition of work.

$$W = F\Delta r \cos\theta$$

$$W = (mg)(h)\cos(0) = mgh$$

In this case, we are using the magnitude of the force and displacement vectors. It doesn't matter if they individually have positive or negative y-components. What matters is that they are both in the same direction such that the work is a positive value.

Now that we have the work, we can look at the change in energy (kinetic energy in this case).

$$W = \Delta K = K_2 - K_1$$

$$3mg = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

In this case, the initial velocity (v_1) is zero. Also, the mass cancels. After a tiny little bit of algebra, we get the following:

$$gh = \frac{1}{2}v_2^2$$

$$v_2 = \sqrt{2gh}$$

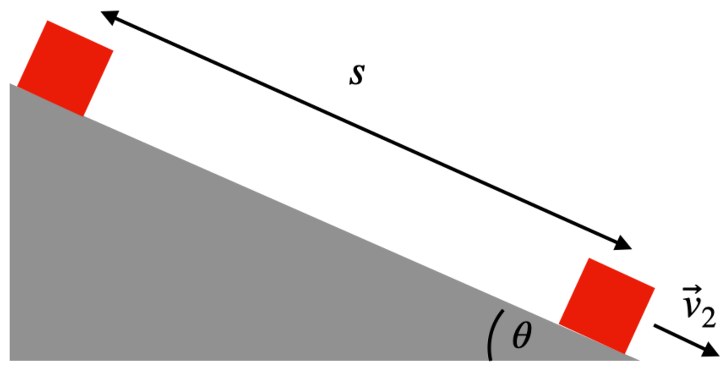
That's it. We can plug in our values (h = 3m) and (g = 9.8 N/kg) to get a final velocity of 7.67 meters per second.

Now for some comments. Notice that I picked y = 0 at the floor. But it doesn't matter. If you have y = 0 m at the starting point, you get a negative value of y = -h for your final position. The change in position is still $\Delta\vec{r} = < 0, -h, 0 >$ meters. It doesn't matter where you place your coordinate system.

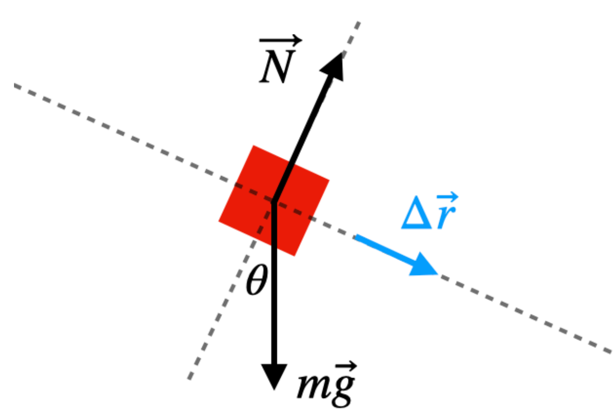
Just for fun, you should solve this same problem using Newton's second law and the kinematic equations. Do it.

BLOCK SLIDING DOWN AN INCLINE

Here's another problem. A block has a mass of m = 100 grams and slides down a ramp that's inclined $\theta = 50$ degrees above the horizontal. The length of the ramp is s = 3.92 meters. What is the speed of the block at the bottom of the ramp?



Again, let's pick a system that's just the block. Now, we need to calculate the work done on this block. That's a little bit more tricky than the previous case. I'm going to start off with a force diagram for the block.



Since there are two forces, we need to consider the work done by each of these individually. Let's start with the normal force. We don't know the magnitude of this force, but we can still write the work as the following:

$$W = N\Delta r \cos\alpha$$

Here α is the angle between the normal force and the displacement. Oh look, $\alpha = \pi/2$ and $\cos(\pi/2) = 0$. Because the normal force is perpendicular to the direction of motion, the work done by this force is zero. That's nice.

What about the work done by the gravitational interaction? We have the force ($m\vec{g}$) and the displacement ($\Delta\vec{r}$)—but what about the angle between these two vectors? If you look at the diagram, maybe you can see that the compliment angle of θ (let's call it θ_c) is the angle between the force and displacement. That means the work done by gravity would be:

$$W = mg\Delta r \cos\theta_c$$

I'm going to make two changes. First, from your trig class (surely you remember trigonometry) we have the following relationship.

$$\cos\theta_c = \cos(\pi/2 - \theta) = \sin\theta$$

Second, the magnitude of the displacement is $\Delta r = s$. This means the work can be:

$$W = mgs\sin\theta$$

Wait! There's something cool here. Look at that diagram above. The right triangle for the incline gives the following: $s\sin\theta = h$. Where h is the starting height of the block.

$$W = mgh$$

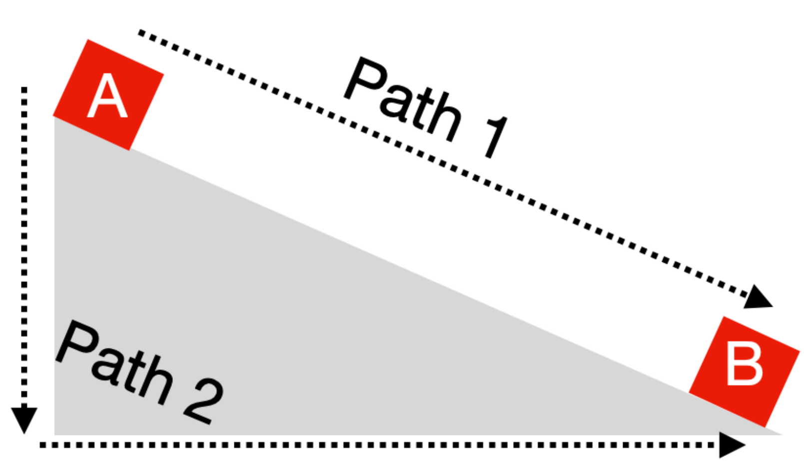
Oh no. Now we are back to the same problem of the falling object. If you crunch the numbers, you might see that I picked an incline angle and length so that $h = 3$ meters—JUST LIKE before.

This means that the block sliding down the incline (without friction) and the dropped ball have the exact same final speed. But there are some differences. The two motions take different times and end up with different vector velocities (the ball is moving straight down and the block is moving at an angle).

The work-energy principle is nice, but it doesn't give us everything. It's a scalar equation so that we don't get the final velocity vector. That's fine. It's still very useful.

CONSERVATIVE FORCES

There was an important reason to look at both the falling ball and the block sliding down a frictionless incline. Imagine that I have an object that goes from point A (at the top) to point B (at the bottom) along two different paths (path 1 and 2).



I just showed that the work done by the gravitational force along path 1 was *mgh*. What about path 2? Well, there are two parts to this path. First the object goes down a distance h with a work of *mgh* (that's the falling ball). Then we have to take a right turn and move horizontally to the final point B. During this second leg, the gravitational force is down but the displacement is horizontal. The angle between the force and displacement is 90 degrees so that the work is zero.

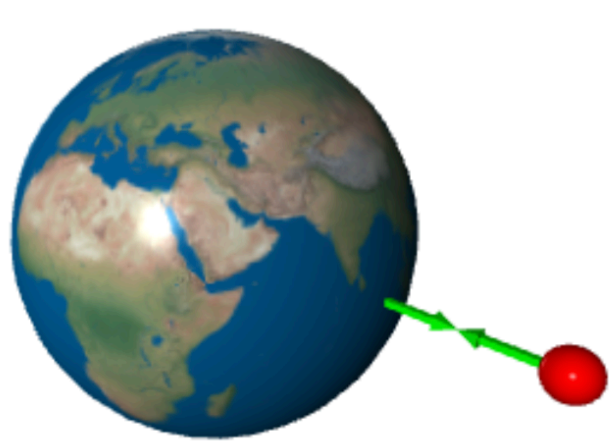
That means that the work done by gravity along these two different paths is the same. Imagine you were able to check EVERY POSSIBLE path between point A and B to calculate the work done by gravity. If you did that, you would find that the work done ONLY depends on the starting and ending points and not the actual path. When the work doesn't depend on the path, we call that force conservative.

Just a couple of comments. Notice that if you made a round trip (like going from A to B and then back to A) the total work would be zero Joules. Also, finding two different paths with the same work doesn't prove the force is conservative—but it's a really nice hint. There's actually a much more robust test for conservative forces, but it's sort of complicated.

Why do we care about conservative forces? These kinds of interactions can be turned into potential energies—with the right choice of a system.

SYSTEMS AND POTENTIAL ENERGY

Let's go back to the dropped ball example. What if instead of choosing just the ball as the system we instead chose the ball plus the Earth as our system. Here's a very dramatic diagram (don't worry about the distances and sizes).



The two green arrows represent the gravitational interaction force pairs (Newton's third law). The Earth pulls on the ball but the ball also pulls on the Earth. So, if we take the Earth-ball as our system we only have an internal interaction (gravity). There are no external interactions with the system so the work has to be zero.

$$W = 0 = \Delta E$$

OK, but we already did this problem. We know that the ball increases in speed so that there must be an increase in kinetic energy. Not only that, but technically this gravitational force acts on the Earth and makes it also increase in speed. That leaves us with the following impossible work-energy equation.

$$W = 0 = \Delta K_{\text{ball}} + \Delta K_{\text{Earth}}$$

This equation is impossible. If both the ball and the Earth increase in kinetic energy, how can the total energy change be equal to zero? The answer is that there must be another type of energy in this system. That energy is the gravitational potential energy. Notice that this gravitational potential is a property of the whole system and not an individual object. It's really a way to represent the gravitational interaction between the two masses—which requires BOTH masses.

Here is how we can define the gravitational potential energy near the surface of the Earth.

$$U_g = mgy$$

Two points. First, it doesn't matter where you measure the value of y from—the only thing that shows up in the work-energy equation is the CHANGE in gravitational potential energy. Second, this assumes that the gravitational force is constant—which is only true if you are near the surface of the Earth. We can look at a better form of the gravitational potential energy in just a little bit.

FALLING BALL WITH EARTH-BALL SYSTEM

Yes. We are doing this again. A ball is dropped from a height of 3 meters above the ground. How fast is it moving just before it hits. This time, we are going to use the system of the ball plus the Earth.

Since there are no external interactions, the work is zero. Instead we have the change in gravitational potential energy. Oh, just for simplicity let's say the change in kinetic energy of the Earth is super tiny (it's fine—the Earth barely moves in this interaction).

$$W = 0 = \Delta K + \Delta U_g$$

For the gravitational potential, we need to pick where the value of y is zero. Just to be different, I'm going to pick the release point as y = 0.

$$0 = K_2 - K_1 + U_{g2} - U_{g1}$$

$$0 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgy_2 - mgy_1$$

In this case, both the initial velocity (v_1) and the initial position (y_1) are equal to zero. That leaves only two non-zero terms.

$$0 = \frac{1}{2}mv_2^2 + mgy_2$$

If the ball falls as distance h, then the final y position would be -h since we picked the initial location as y = 0.

$$0 = \frac{1}{2}mv_2^2 + mg(-h)$$

$$v_2 = \sqrt{2gh}$$

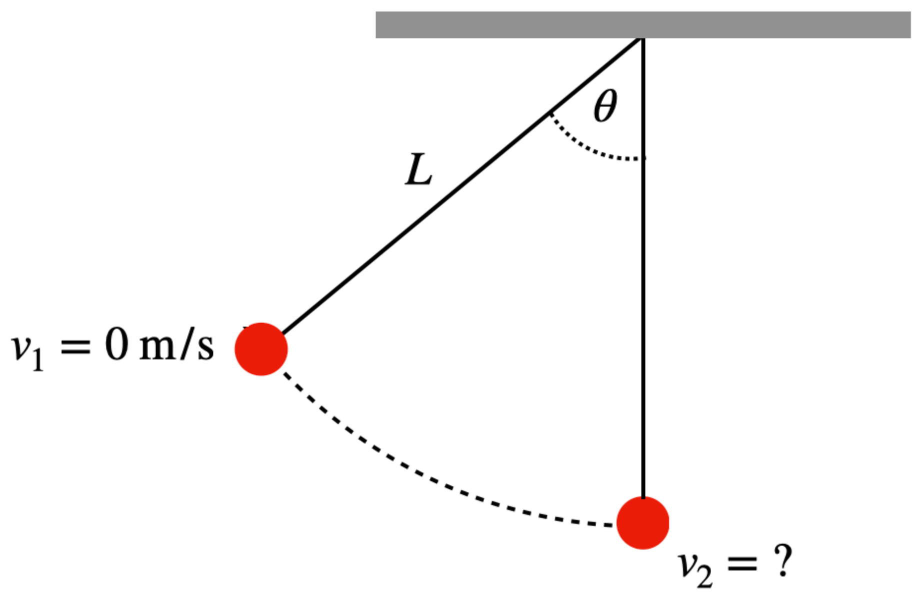
I skipped some steps at the end—but come on, it's the same problem as before. It shouldn't be surprising that we get the same final velocity. That final velocity doesn't depend on how we solve the problem.

One important thing to notice. For this problem, the change in gravitational potential energy was $-mgh$. In the case where we used the work done by gravity, it was mgh. So, if you want to potential energy from a force, it's just the negative of the work done by that force.

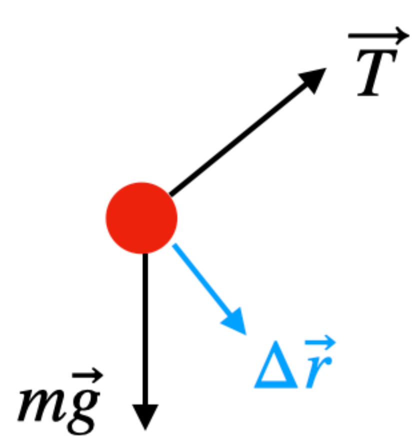
SWINGING MASS ON A STRING

For falling objects, does it matter what you pick as your system? Is it just personal preference whether it's the object by itself or the object plus the Earth? Yes, you can do it either way. However, if you are not careful you will get yourself into situations in which the work done by gravity is quite challenging to calculate. Consider the following example.

A 100 gram mass (m) is connected to a 0.5 meter long string (length L). The mass is pulled back at an angle of $\theta = 50^\circ$ and released from rest. How fast will the mass be moving when it reaches its lowest point?



Suppose we pick just the ball as our system. In that case, we have two force that we need to consider for the work done. There is the downward pulling gravitational force and the tension from the string.



Let's start with the work done by the tension force. This *could* be very tricky. Remember that there is no single equation for the tension from a string. It's not something you can just calculate like the gravitational force. Instead, the tension is a force of constraint. This means that it's magnitude will be whatever it needs to be to keep that mass at the same distance from the pivot point. It constrains the motion of the ball to move in a circle. So, this force will change in both magnitude and direction and seems like it would be impossible (or at least challenging) to calculate the work.

But wait! We have a trick. Although the direction of the tension force changes direction, it's always perpendicular to the direction of travel since the object is moving in a circle. This gives the following for the work done by the tension.

$$W_T = T\Delta r\cos(\pi/2) = 0$$

It's just zero. That's nice. OK, what about the work done by the gravitational force? Yes, this is a constant downward force. But the problem is now Δr —the displacement is not a single vector. Instead, the path is continually changing direction. How do you calculate the work done by a force around a curve? Yes, there is a way to do this—but why do something complicated when we don't have to. Right?

Instead of just choosing the ball as our system, let's use the ball plus the Earth. In that case, we don't calculate the work done by gravity but instead have the change in gravitational potential energy. Problem avoided.

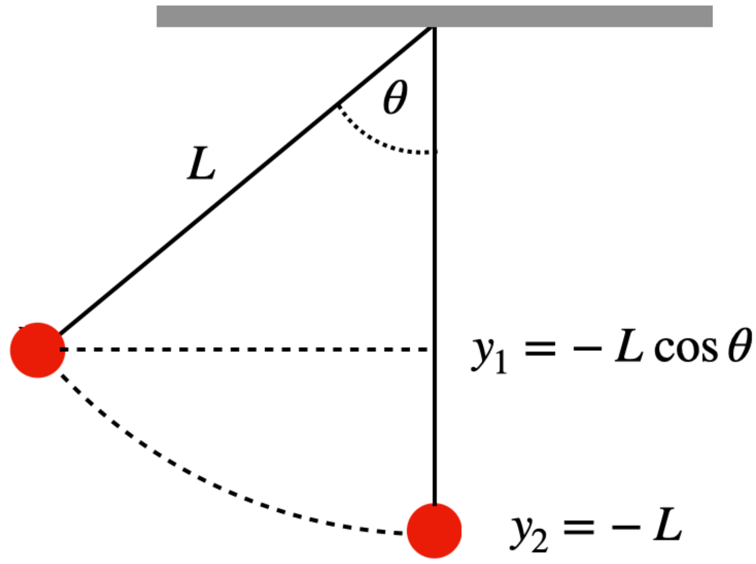
In this system we still have the work done by the tension as zero. Yes, you could say that the string is part of the Earth—but it's better to keep it separate. The work is zero so it doesn't really matter either way. Now we have the following work-energy equation:

$$W = 0 = \Delta K + \Delta U_g$$

$$0 = K_2 - K_1 + U_{g2} - U_{g1}$$

The next step is to pick the location where $y = 0$. Just like the dropping ball problem, there is no wrong choice here—but there is a better choice. Just because I have lots of experience with this kind of stuff, I'm going to pick the pivot point (where the string attaches to the top) as $y = 0$. That means that the final position of the mass will be straight down at $y_2 = -L$ (the length of the string).

But what about the y position at location 1? We need a trick. Here's diagram.



If we draw a horizontal line from position 1 to a vertical line underneath the pivot point then we create a right triangle. The hypotenuse of this triangle is L and the adjacent side would be the negative position 1.

Putting this all together, we get:

$$0 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mg(-L) - mg(-L\cos\theta)$$

The initial velocity is zero.

$$0 = \frac{1}{2}mv_2^2 - mgL + mgL\cos\theta$$

$$0 = \frac{1}{2}mv_2^2 - mgL(1 - \cos\theta)$$

$$v_2 = \sqrt{2gL(1 - \cos\theta)}$$

There you go. The final velocity.

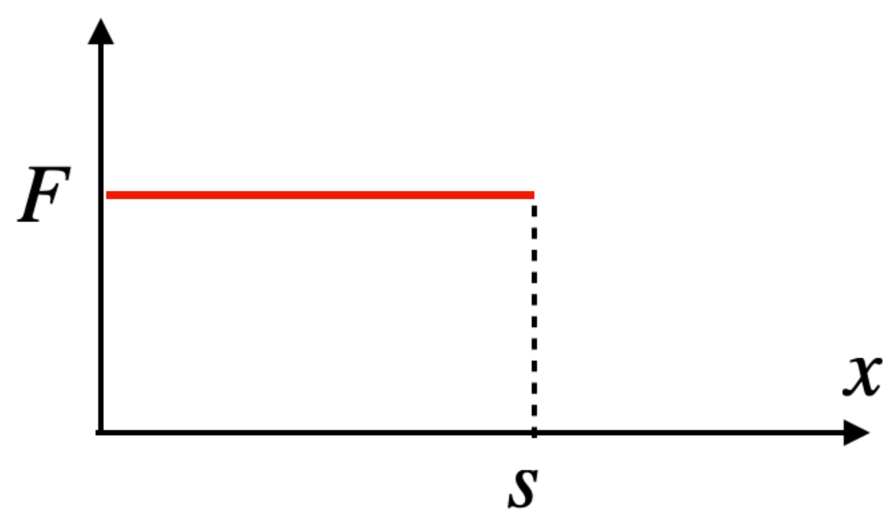
SPRING POTENTIAL ENERGY

Remember Hooke's Law. This says that when you stretch or compress a spring, the spring exerts of a force that's proportional to stretch (or compression). We can write the magnitude of this spring force as:

$F_s = ks$

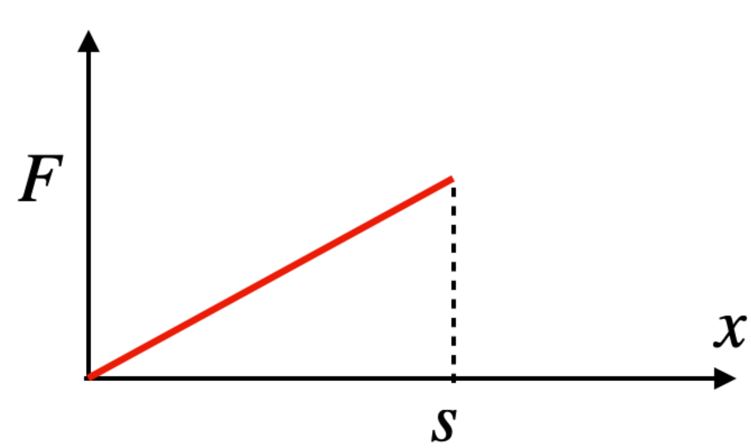
Here k is the spring constant—a measure of the stiffness of the spring and s is the stretch. It's a scalar equation, so don't put the negative sign there. But this is not about forces, we are talking about work.

We can figure out the work done by a spring by thinking about a different situation. Imagine I apply a constant force to an object and push it from $x = 0$ to $x = s$ (in the x-direction). If you made a plot of F vs. x, it would look like this.



Since the force is constant, the work would just be $W = Fs$. Simple. But look—from the graph F multiplied by s is also the area under that force line. That might be useful.

What if we repeat this process with a spring. Here's the applied force as a function of distance for a spring (assuming it's unstretched at x = 0).



Now we have a triangle and the work done by this force will just be the area under the curve.

$$W = \frac{1}{2}Fs$$

But at the stretch distance of s, the force is $F = ks$. Putting that in for F:

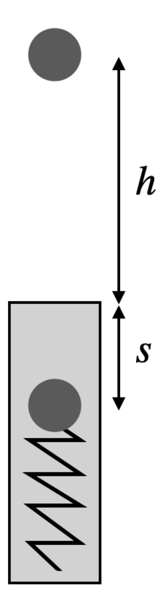
$$W = \frac{1}{2}(ks)s = \frac{1}{2}ks^2$$

That's the work done to stretch the spring—but the spring would exert a force in the opposite direction so the work done by the spring would be the negative of that $(-\frac{1}{2}ks^2)$. If we want to make this into spring potential energy, we get the negative of a negative.

$$U_s = \frac{1}{2}ks^2$$

PROJECTILE LAUNCHER

Many physics labs use the spring powered ball launchers to study projectile motion. Suppose we have a launcher with a 25 gram ball. The launcher has a spring constant of 144 N/m and is compressed 2 cm. The ball is then launched straight up. How high does it go?



Of course we should choose the spring, ball, and the Earth as our system. In that case, there are no external interactions so that the work is zero.

$$W = \Delta K + \Delta U_g + \Delta U_s = 0$$

$$0 = K_2 - K_1 + U_{g2} - U_{g1} + U_{s2} - U_{s1}$$

Let's pick the starting position of the ball as $y = 0$. Notice that at both the starting and ending position (the highest point) the velocities are zero. Even better, the initial gravitational potential energy is zero (starts at $y = 0$) and the final spring potential energy is zero (no longer interacting with the spring).

$$0 = mg(h + s) + 0 - \frac{1}{2}ks^2$$

Now we can solve for h.

$$\frac{1}{2}ks^2 = mg(h + s) = mgh + mgs$$

$$mgh = \frac{1}{2}ks^2 - mgs$$

$$h = \frac{ks^2}{2gm} - s$$

Putting in our values, we get a ball height of 9.8 centimeters. That's not very high, but it's good enough for a physics lab.

REAL GRAVITATIONAL POTENTIAL ENERGY

For our previous gravitational potential energy, we assumed the gravitational force was a constant ($m\vec{g}$). However, we know (probably from a previous chapter that I haven't yet created) that there is a better model for the gravitational force between two masses. The magnitude of this force can be expressed as:

$$F = G\frac{m_1 m_2}{r^2}$$

Where m_1 and m_2 are the masses as r is the distance between the centers of the two objects. G is the gravitational constant ($G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$). But the main point is that this force decreases as r increases. What we took a mass (m_1) at an infinite distance from another stationary mass (m_2) and then moved it closer. Looking at mass m_1, it starts off with a zero gravitational force (since $r = \infty$). As it moves closer, the gravitational force does work on it, but that work increases as the masses get closer. Since the gravitational force is in the same direction as the motion, the work done by the gravitational force would be positive.

Using some cool math, it turns out the work going from infinity to some distance r from mass 2 would be:

$$W = G\frac{m_1 m_2}{r}$$

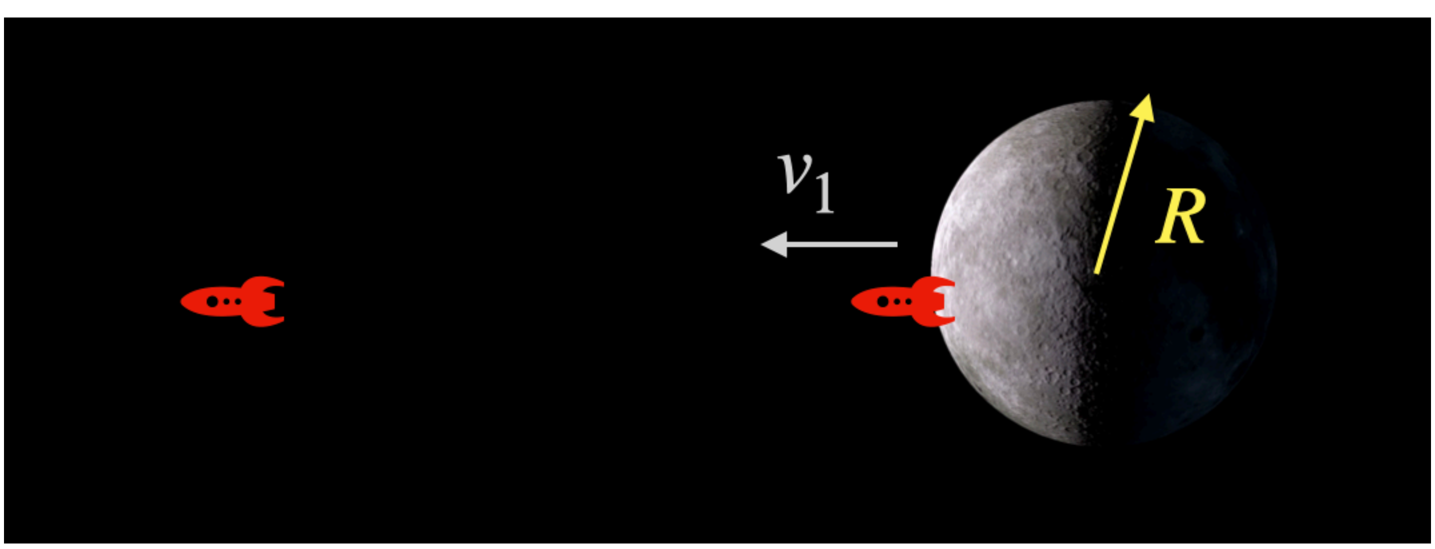
Now suppose we want to include both objects in our system and convert this to a potential energy. In that case the potential is the negative of the work done by gravity.

$$U_G = -G\frac{m_1 m_2}{r}$$

It might seem weird that the gravitational potential energy is negative—but it's cool, trust me. This is what we have for an attractive force that turns into a potential. Notice that I'm using U_G for the gravitational potential energy and U_g for the potential near the surface of the Earth.

ESCAPE VELOCITY

Suppose you have a spacecraft on the surface of the moon (where there's no air—that makes things nice). The spacecraft has rockets that will fire for a very short time to give the ship an initial velocity. The spacecraft will then move up without any thrust. Question: how fast does the ship need to initially be traveling so that it goes up and never comes down? Since the ship will "escape" the gravitational interaction of the moon, we call this the escape velocity.



For the system, let's go with the rocket plus the moon. That means there's no external work (right after the rockets were fired). We now have the following the work-energy equation.

$$W = \Delta K + \Delta U_G = 0$$

$$0 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + -G\frac{M_m m}{r_2} + G\frac{M_m m}{r_1}$$

OK, we have some stuff going on here. I'm using M_m to represent the mass of the moon. Are any of these terms equal to zero? Yes. If we just barely escape the moon, the final velocity (v_2) is zero and the final position is $r_2 = \infty$. Remember that if you divide by a very large number, you get a very small number. So, if you kind of divide by infinity, you kind of get zero. Please don't tell your math friends, they might have a freak out fit—but it's fine for us. That means there are two terms that are zero.

Finally, the initial value of r is R (the radius of the moon), we get the following.

$$0 = -\frac{1}{2}mv_1^2 + G\frac{M_m m}{R}$$

$$v_1^2 = \frac{2GM_m}{R}$$

$$v_1 = \sqrt{\frac{2GM_m}{R}}$$

There's your escape velocity. The moon has a mass of 7.35×10^{22} kg and a radius of 1.74×10^6 m. Plugging in these values (the mass of the spacecraft canceled in case you didn't notice) we get 2374 meters per second. So, if you launch faster than this velocity the spacecraft will escape and with some left over kinetic energy. If you launch with a speed lower than this, it will rise up but then fall back down.

Notice that you can repeat this calculation for any gravitational interaction. The escape velocity depends on mass of the object you are escaping AND the starting position. You could use this for the escape velocity from the solar system based on the Earth's distance from the sun.

Bonus: if you go backwards and say that the escape velocity is the speed of light (3×10^8 m/s) you can use that to find the radius of the sun at which even light can not escape. This is called the Schwarzschild radius—it's one of the first ways that we estimated the size of a black hole.

OTHER ENERGY

So far, we have the following forms of energy (depending on the choice of the system):

- Kinetic energy: $\frac{1}{2}mv^2$

- Gravitational potential energy (surface) mgy
- Spring potential energy $\frac{1}{2}ks^2$
- Better gravitational potential energy $-G\frac{m_1 m_2}{r}$

But are there other types of energy? Yup. There are two that you might see in an introductory physics course.

The first is thermal energy. When an object warms up, it increases in temperature. We can calculate this change in energy as:

$$\Delta E_T = mC\Delta T$$

Here m is the mass of the object and ΔT is the change in temperature (use units of Celsius or Kelvin). The C represents the specific heat capacity. This is a value that depends on the type of material. We also have a change in energy for a material that goes through a phase change. For solid to liquid, this energy change is:

$$\Delta E_f = mL_f$$

Where L_f is the latent heat of fusion. There's also the latent heat of vaporization (L_v)for when a material goes from liquid to a gas.

$$\Delta E_v = mL_v$$

POWER

If you take a textbook from the floor and put it on a table, that takes about 10 Joules of energy. But what's the difference between doing that in 1 second or 10 seconds? The change in energy is the same but the time-rate of change of energy is different. We define this time rate of change for energy as the power.

$$P = \frac{\Delta E}{\Delta t}$$

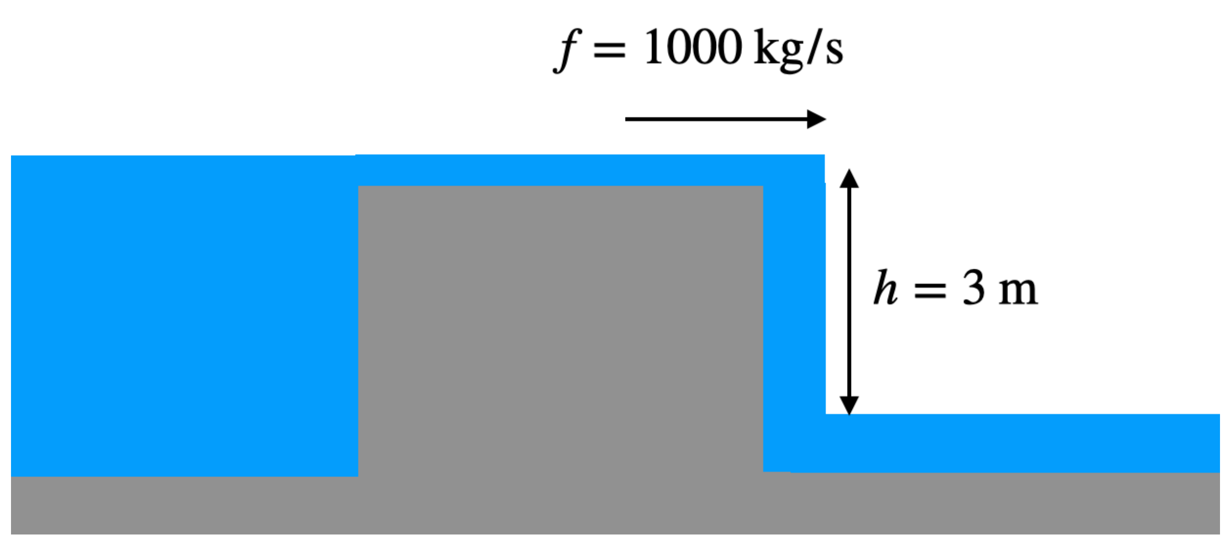
If the energy is measured in Joules and the time interval in seconds, the the power would be in Joules per second which is called a Watt.

When you look at large batteries, they will often be rated in kWh - kilowatt hours. Just to be clear, this is a unit of energy and not power. Remember that power multiplied by a time interval would be energy.

How about a power problem?

HYDROELECTRIC POWER

Imagine you have a dam on a river used to generate electrical power. The river has a flow rate of 1000 kilograms per second and the height of the damn is 3 meters. What is the maximum power output for this hydroelectric generator?



Yes, that's my diagram for a dam. Suppose we have a time interval Δt. How much mass flows over the dam in this time?

$$m = f\Delta t$$

Where f is the flow rate in kg/s. We can find the change in energy of this water. I'm going to assume it falls and gains in kinetic energy—but then that kinetic energy is converted into some type of electrical energy. So, we just want the change in gravitational potential energy.

$$\Delta E = mg\Delta y = (f\Delta t)gh$$

Now we can calculate the power by dividing by the time interval.

$$P = \frac{\Delta E}{\Delta t} = \frac{fgh\Delta t}{\Delta t} = fgh$$

Let's just do a quick check on the units.

$$\text{units} = \left(\frac{kg}{s}\right)\left(\frac{N}{kg}\right)(m) = \frac{Nm}{s}$$

A Newton-meter is a Joule, so this does indeed give us Joules per second (watts). Plugging in the numerical values I get a power output of 29.4 kiloWatts. Nice.