Velocity Dependent Forces

1. Suppose you have a ball (mass m) moving horizontally with an initial velocity of $v_0 = 2$ m/s in a region in which there is a drag force such that in the x-direction, $F_D = -bv$ where b is a drag coefficient. Describe the motion of the ball. Make a sketch of both velocity vs. time and position vs. time (just a sketch) - don't worry about the numbers.

2. Draw a force diagram for the ball. Write down Newton's 2nd law in the x-direction. Express the acceleration as $\frac{dv}{dt}$.

3. This differential equation is separable. Get all the "v" terms on one side of the equation and "t" terms on the other side. Integrate both sides (don't forget the integration constant).

4. Use the initial conditions ($v(0) = v_0$ an $x(0) = x_0$) to determine the integration constant.

5. Write $v(t)$ as $\frac{dx}{dt}$. Use this in the differential equation to find an equation of motion. Again, don't forget the integration constant and the initial conditions.

6. Describe a method to create a numerical model to determine the motion of the ball (as though someone was creating code in python).

7. Suppose you take this same ball and drop it from some starting vertical position y_0. Draw a force diagram for this situation (yes, including the gravitational force). Sketch a velocity-time graph.

8. Eventually the drag force will equal the gravitational force and the net force will be zero. We call this terminal velocity (v_T). Write an expression for the terminal velocity.

9. TRICK. Write Newton's second law for this case but put the positive y direction DOWN (yes, it's just a trick). Again, use $\frac{dv}{dt}$ for the acceleration.

10. Trick number 2. In your equation for Newton's second law, represent the gravitational force (mg) in terms of the terminal velocity. With some algebra, you should be able to get a term that looks like $(v - v_T)$ and we can use a substitution that $u = v - v_T$. Now rewrite the differential equation in terms of u.

11. Separate variables and integrate both sides. To get the integration constant, assume that $u(0) = u_0$. You should get an equation for u(t). Now substitute for u to get it back in terms of v.

12. Can you use $v = \frac{dy}{dt}$ to integrate and get y(t)? Yes, you can.

13. Imagine that instead of dropping a ball, it's thrown horizontally. Draw a free body diagram. Is it possible to have two independent equations for Newton's second law?