Accelerated Reference Frames

1. An elevator is at rest. A person inside tosses a ball straight up with an initial velocity of 1 m/s. How high does the ball travel? How long does it take the ball to get back to the starting position?

2. Now the elevator is traveling upward with a constant velocity of 2 m/s. The ball is thrown up with a velocity of 1 m/s (with respect to the elevator). How high does the ball go with respect to an external (non-moving) reference frame? How high is the ball (with respect to stationary frame) when the ball is caught? How long was it in the air?

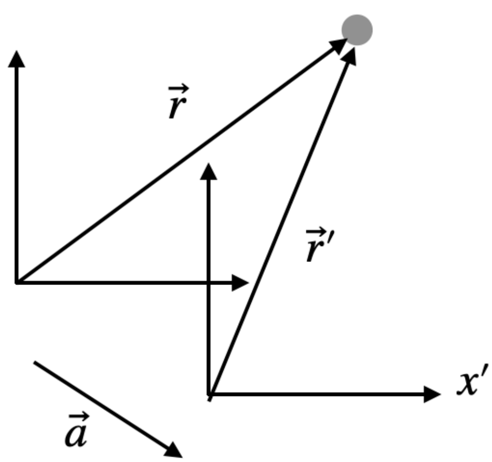
3. Now the elevator starts from rest, but has an upward acceleration of 2 m/s^2. The ball is thrown upward with a 6 m/s with respect to the elevator. How high does it travel with respect to the elevator? How long is it in the air?

4. We can also look at this same problem by adding a fake force. The fake force will take the form:

   $\vec{F}_{\text{fake}} = -m\vec{a}_{\text{frame}}$

   Where a-frame is the acceleration of the reference frame. Redo the problem using a fake force.

5. Just to be formal, let's say that we have two reference frames, one with x and y and one with x' and y'. The prime frame is accelerating with a value a.

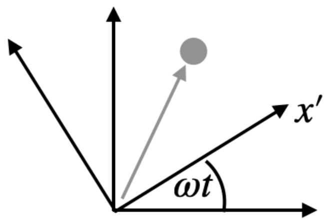


We can find the location of an object in the two frames ($\vec{r}$ and $\vec{r}'$). In this case, we have the following relationship.

$\vec{r}' = \vec{r} + \frac{1}{2}\vec{a}t^2$ (assuming the two frames are in the same position with the same velocity at t = 0 seconds.

If that is the position, find an expression for the velocity in the prime coordinates (take the time derivative). Take the time derivative again to get the acceleration and show that you can write Newton's second law as the mass times acceleration plus a fake force.

6. What about a rotating reference frame? Imagine that you have the prime frame sharing the origin with a stationary frame but the prime frame rotates with an angular velocity ω.



In this case, it's going to be easier to use polar coordinates. Write the acceleration vector ($\ddot{\vec{r}}$) in polar coordinates. You might need to look this up, but you have used it before. Remember?

In the stationary frame, an object will have the coordinates (r, θ). Since the prime frame has the same origin but rotated, the object will have coordinates $(r', \theta') = (r, \theta - \omega t)$. What is the value of $\dot{\theta}'$ in terms of $\dot{\theta}$?

We can now write the acceleration vector in the prime coordinates by replacing $\dot{\theta}'$ with $\dot{\theta} - \omega$. Write the new acceleration vector.

Group all the terms that look like acceleration in the stationary frame. You should be able to show (with some algebra):

$$\ddot{\vec{r}}' = \ddot{\vec{r}} + 2\omega(r\dot{\theta}\hat{r} - \dot{r}th\hat{e}ta) - r\omega^2\hat{r}$$

We can call these TWO fake forces:

$$\vec{F}_{\text{coriolis}} = -2m\omega(r\dot{\theta}\hat{r} - \dot{r}th\hat{e}ta)$$

And

$$\vec{F}_{\text{centrifugal}} = mr\omega^2\hat{r}$$

7. We can also write these fake forces as:

$\vec{F}_{\text{coriolis}} = -2m\vec{\Omega} \times \vec{v}$ (where $\vec{\Omega}$ is the angular velocity vector of the rotating frame.

$\vec{F}_{\text{centrifugal}} = m(\vec{\Omega} \times \vec{r}) \times \vec{\Omega}$

Suppose we use cylindrical coordinates and let our frame rotate such that:

$\vec{\Omega} = \omega \hat{z}$

If the velocity (in polar coordinates) is:

$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$

Show that these two formulas produce the same result.